Lecture 2 - Motion in one dimension
Covers Sections 2.1 - 2.4

Mechanics is the study of motion (kinematics) and of the forces which produce motion (dynamics). We shall first study kinematics in one dimension. To describe kinematics in one dimension, we will introduce several different quantities. We all have a colloquial understanding of terms like speed, velocity, acceleration, distance. The physics usage of these terms is similar to the colloquial usage, but there are important differences. These differences will become very pronounced when we study motion in two dimensions.

The quantities we need to understand and to calculate with are:
(i) Position, x, displacement $\Delta x_{if} = x_f - x_i$, distance d (ii) Average velocity $\overline{v}$, instantaneous velocity v, average speed, instantaneous speed. (iii) Average acceleration $\overline{a}$ and instantaneous acceleration a.

A *scalar* quantity is just a number and has no direction. A *vector* quantity has a magnitude and a direction. Distance and speed are scalars, while position, velocity and acceleration are vectors.

Position, velocity and acceleration

To define a position we have to first decide on our reference point or origin. In general we have to define a reference frame, as we will see in the case of two dimensional motion.

The position of point P is an arrow which points from the origin to P. This arrow has a magnitude, and a direction, which is either positive or negative in this one dimensional case. The displacement between two positions P and Q is an arrow which points from P to Q, and we define $\Delta x_{PQ} = x_Q - x_P$. For example if $x_P = 10$ and $x_Q = 12$, $\Delta x_{PQ} = x_Q - x_P = 2$. Notice that if $x_P = 10$ and $x_Q = 8$, $\Delta x_{PQ} = -2$, indicating that the arrow points in the negative direction.

In kinematics, we are interested in motion and a useful way of looking at motion is through motion diagrams or motion equations, which show position (and/or velocity and acceleration) as a function of time. For example consider motion described by,

$$x = -3 + 4t - t^2 \tag{1}$$

where x is in meters and t is in seconds. Note that to ensure that this equation is dimensionally correct, the units of the number -3 are meters, the units of the number 4 are m/s and the units of the -1 prefactor of the t^2 term

are m/s^2. When motion equations are written down these units are usually omited, but they are implied and sometimes you will be asked to figure out what the units must be to ensure that the equation is dimensionally correct.

We can draw many different graphs of x vs t. Are all of them physically reasonable? One restriction is that at any given time, the position is single valued because we can't be in two places at the same time. However we can return to the same position at a later time, as occurs in the motion described by Eq. (1). From motion diagrams or motion equations, we can find the displacement between two times, t_i and t_f, so that $\Delta x_{if} = x(t_f) - x(t_i)$.

Now we are ready to talk about *velocity*. Velocity is a vector so it can also be thought of as an arrow which has a magnitude and a direction. The average velocity is the rate of change of position with time. In mathematical terms this is,

$$\overline{v} = \frac{x(t_f) - x(t_i)}{t_f - t_i} = \frac{\Delta x_{if}}{\Delta t_{if}} \tag{2}$$

where we have defined $\Delta t_{if} = t_f - t_i$. The instantaneous velocity is the special case where Δt_{if} becomes very small. In fact in the limit where this quantity goes to zero is the limit in which the ratio $\Delta x_{if}/\Delta t_{if}$ becomes a derivative and we enter the world of calculus. Though we shall not use calculus, we are carrying out similar calculations when we calculate the instantaneous velocity. Lets look at the motion diagram corresponding to Eq. (1). Some points to note:

(i) The average velocity in going from B to D is zero!

(ii) The average velocity in going from A to B is $3m/s$

(iii) The average velocity in going from C to D is $-1m/s$

(iv) The instantaneous velocity at any position is the slope of the graph at that position.

The *acceleration* is the rate of change of velocity and it is also a vector so it can be thought of as an arrow with a magnitude and a direction. In mathematical terms the average acceleration is then given by,

$$\overline{a} = \frac{v(t_f) - v(t_i)}{t_f - t_i} = \frac{\Delta v_{if}}{\Delta t_{if}} \tag{3}$$

where $v(t)$ is the instantaneous velocity at time t and we use $\Delta t_{if} = t_f - t_i$, as in Eq. (2). The instantaneous acceleration is the special case where Δt_{if} becomes very small. Looking at the motion curve corresponding to Eq. (1), note the following:

(i) The acceleration is the *change in the slope* of the motion curve.

(ii) The change in slope is negative for the whole motion curve, which means that Eq. (1) describes a system which is accelerating in the negative x direction.

(iii) Draw a graph and/or write an equation for the case where the change in slope is positive?

Distance and speed

As remarked above, the distance and speed are scalars. To be frank, there is quite of bit of confusion about the definition of these quantities, even in introductory textbooks. The case where confusion arises is illustrated by the example Quiz 2.1 in the text (page 27). The total displacement is zero, however the total distance is 200 yards. In our calculations, we will take the distance to be the total distance travelled. Note that the magnitude of the displacement is the *net distance* which in this example is zero. In a similar way we shall define the average speed as follows,

$$average\ speed = \frac{total\ distance}{total\ time} \qquad (4)$$

Note the following about distance and speed

(i) The distance is taken to be the total distance travelled

(ii) Distance and speed are always positive

(iii) The instantaneous speed is the modulus or magnitude of the instantaneous velocity.

(iv) The average speed is always greater than or equal to the modulus of the average velocity, over the same time interval.

Example

Considering running from the origin (ie. $x_i = 0$) directly to a position $x = 25m$ and then reversing direction and running back to the final position $x_f = 11$ in a total time of $\Delta t_{if} = 3s$. The displacment $\Delta x_{if} = 11$, so the average velocity $\overline{v}_{if} = 3.67m/s$, but the total distance travelled is $39m$ so the average speed is $39m/3s = 13m/s$.